**Chapter 6**

**Applications of Integration**

**6.6 Moments and Centers of Mass**

**Section Exercises**

**For the following exercises, calculate the center of mass for the collection of masses given.**

255.  at  and  at

Answer: 

257. Unit masses at 

Answer: 

259.  at  and  at 

Answer: 

**For the following exercises, compute the center of mass **

261.  for 

Answer: 

263.  for 

Answer: 

265.  for 

Answer: 

267.  for 

Answer: 

269.  for 

Answer: 

**For the following exercises, compute the center of mass. Use symmetry to help locate the center of mass whenever possible.**

271.  in the triangle with vertices , , and 

Answer: 

**For the following exercises, use a calculator to draw the region, then compute the center of mass. Use symmetry to help locate the center of mass whenever possible.**

273. **[T]** The region bounded by, , and 

Answer: 

275. **[T]** The region between  and 

Answer: 

277. **[T]** The region bounded by , 

Answer: 

279. **[T]** The region bounded by and  in the first quadrant

Answer: 

**For the following exercises, use the theorem of Pappus to determine the volume of the shape.**

281. Rotating  around the-axis between  and 

Answer: 

283. A general cylinder created by rotating a rectangle with vertices , , andaround the-axis. Does your answer agree with the volume of a cylinder?

Answer: 

**For the following exercises, use a calculator to draw the region enclosed by the curve. Find the area and the centroid  for the given shapes. Use symmetry to help locate the center of mass whenever possible.**

285. **[T]** Quarter-circle: , , and 

Answer: 

287. **[T]** Lens:  and 

Answer: 

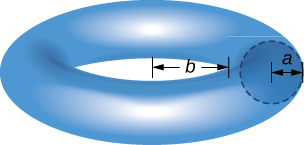
289. **[T]** Half-ring: , , and 

Answer: 

291. Find the generalized center of mass between , , and. Then, use the Pappus theorem to find the volume of the solid generated when revolving around the *y*-axis.

Answer: Center of mass: , volume: 

293. Use the theorem of Pappus to find the volume of a torus (pictured here). Assume that a disk of radius  is positioned with the left end of the circle at , and is rotated around the *y*-axis.



Answer: Volume: 

**Student Project**

**The Grand Canyon Skywalk**

1. Compute the area of each of the three sub-regions. Note that the areas of regions  and  should include the areas of the legs only, not the open space between them. Round answers to the nearest square foot.

Answer: Let  denote the area of  Then



Let  denote the area of  Then



Let  denote the area of  Then



3. Calculate the center of mass of each of the three sub-regions.

Answer: Let , , and  denote the centers of mass of the three regions. We note that all three regions are symmetric with respect to the *y*-axis. Thus



Additionally,  and  are symmetric vertically, so



To find , we need to find the moment with respect to the *x*-axis, . Looking at , we see it is bounded above by



and below by



Then



Therefore we have



In summary, we have



5. Assume the visitor center weighs 2,200,000 lb, with a center of mass corresponding to the center of mass of  Treating the visitor center as a point mass, recalculate the center of mass of the system. How does the center of mass change?

Answer: The visitors’ center weighs 2.2 million pounds, so its mass, , is given by



We know , so we have . Let  denote the center of mass of the system. Then, as before, by symmetry, , and we need to find .

Then the mass of the system, , is given by



Thus,



Then 

In summary,



Notice that the center of mass is now on land, about 10 feet from the edge of the canyon.

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